Development of Stratonis: A Study of Stratified, Layered Structures in Mathematics

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Introduction

Stratonis is a mathematical field dedicated to the study of stratified, layered structures within various mathematical contexts. This field encompasses the analysis, modeling, and applications of such structures in abstract spaces. The layers can represent different levels of complexity, dimensionality, or properties, and their interactions are of particular interest.

Notation

Let $\mathcal{S}$ denote a stratified space. A stratified space can be thought of as a union of disjoint strata S_i, each of which is a manifold.

$$\mathcal{S} = \bigcup_{i \in I} S_i$$

Each S_i is a d_i-dimensional manifold.

Stratification Maps

Define a stratification map $\sigma : \mathcal{S} \to \mathbb{Z}$ which assigns a dimension to each point in $\mathcal{S}$.

$$\sigma(x) = \dim(S_i) \quad \text{if} \quad x \in S_i$$

Stratified Differential Forms

A differential form on a stratified space $\mathcal{S}$ can be restricted to each stratum.

$$\Omega^k(\mathcal{S})|_{S_i} = \Omega^k(S_i)$$

Stratified Homology and Cohomology

Define the homology and cohomology groups of a stratified space:

$$H_k(\mathcal{S}) = \bigoplus_{i \in I} H_k(S_i)$$

$$H^k(\mathcal{S}) = \bigoplus_{i \in I} H^k(S_i)$$

Stratified Morse Theory

Consider a smooth function $f : \mathcal{S} \to \mathbb{R}$. The critical points of f can be analyzed within each stratum.

$$\mathrm{Crit}(f) = \bigcup_{i \in I} \mathrm{Crit}(f|_{S_i})$$

The Morse index $\lambda(x)$ at a critical point $x \in S_i$ is given by the usual definition restricted to S_i.

Stratified Geometry

Define the metric properties of stratified spaces: A Riemannian metric g on $\mathcal{S}$ induces a metric on each stratum g_i on S_i.

Stratified Dynamics

Consider a dynamical system on $\mathcal{S}$ described by:

$$\dot{x} = F(x)$$

where $F : \mathcal{S} \to T\mathcal{S}$ respects the stratification, i.e., $F(S_i) \subseteq TS_i$.

Formulas and Properties

Stratified Volume

The volume of a stratified space can be computed as:

$$\mathrm{Vol}(\mathcal{S}) = \sum_{i \in I} \mathrm{Vol}(S_i)$$

Stratified Curvature

The curvature of each stratum S_i can be considered, and the total curvature of $\mathcal{S}$ is:

$$\mathrm{Curv}(\mathcal{S}) = \sum_{i \in I} \int_{S_i} K_i \, d\mathrm{vol}_i$$

where K_i is the sectional curvature of S_i.

Stratified Laplacian

The Laplace operator on a stratified space is defined piecewise:

$$\Delta_{\mathcal{S}} f = \bigoplus_{i \in I} \Delta_{S_i} f|_{S_i}$$

Stratified Heat Equation

The heat equation on $\mathcal{S}$ is given by:

$$\frac{\partial u}{\partial t} = \Delta_{\mathcal{S}} u$$

Stratified Hamiltonian Dynamics

The Hamiltonian $H : T^*\mathcal{S} \to \mathbb{R}$ defines the dynamics:

$$\dot{q} = \frac{\partial H}{\partial p}, \quad \dot{p} = -\frac{\partial H}{\partial q}$$

Conclusion

Stratonis provides a rigorous framework for studying layered structures in mathematics. By analyzing the properties and dynamics of stratified spaces, we can uncover new insights into the behavior of complex systems and their interactions. The notations and formulas introduced here serve as foundational tools for further exploration and development in this field.

References

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